

FTL Travel
And The Math Behind What It Can Do

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IB Math SL 1

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Introduction:

Indeed, the speed of light (also known as C) is measured to be approximately 299,792,458 meters per second. That's quite a high number and is the fastest a physical substance can travel. The substance that may travel this fast is called a photon, its also known as light. We as a species have never observed anything ever going faster through space than light, except for space itself, but that doesn't really count as its not moving through space. Over the previous summer, I went up to my local university (the University of Utah) and spend a week taking a course on astronomy. And something went off in my head and I've been even more fascinated by astronomy and its related physics ever since. I have a little previous knowledge about FTL travel from some videos I have seen on YouTube, and the entire theory of it being possible stated off pretty much as a Star Trek joke, so much so that the proposed design for a Warp Drive: ship by NASA is called the Enterprise. Named after the Star Trek ship. I'm not a huge Star Trek fan myself, but I thought that was hilarious and I loved it.

I really love the idea of FTL travel, I play video games pretty heavily, or at least used to before my massive workload of IB took control of my time almost completely. There was one game that I was extremely looking forward to called No Man's Sky, This was a video game about exploring the universe and discovering cool new worlds. With my previous interest in astronomy like when I would look forward for days to go to the Clark Planetarium to see a new documentary about Black Holes or Astro-Chemistry fed into this excitement. This of course, was only a game. I've always dreamt of exploring new worlds in person, really experiencing alien geography and potentially ecosystems. This would only be possible with FTL travel due to the extreme vastness and spaciousness of space. This is the only way besides teleportation which has even more problems around it, that my dreams of conquering the stars quickly could come true.

FTL travel is also really important because if we discover it is physically possible, it would completely change everything we thought was possible, we should be able to reach anywhere in the universe potentially almost instantly to an observer on the earth. It's the simplest way for the human species to establish a galactic empire, and potentially one that expands much further into the universe than just the galaxy. The worlds it could unlock and the new ways we could look at physics would be astounding. IT would be the invention of the century easily and would mark a huge step in our evolution as a civilization. It could let us search for alien life and let us take chemical samples from places all around the universe for analysis.

Gravity is the distortion of space time according to Einstein's relativity, which is the best theory we have about all that 'everything' jazz going on today. The force of gravity itself is quite interesting because it is what allows this whole thing anyways. Gravity is cool here because it would allow for infinite acceleration if the gravitational disturbance pulling/pushing the spaceship was also moving with it at the same speed.

Aim of the Exploration:

The Aim of this exploration is to determine the perfect values to fit into the newton gravity equation to see how far away the gravitational forces/anomalies would have to be from the ship, how powerful they would have to be to reach certain accelerations/speeds, and to determine the gravitational force they would give off with these conditions. Also, how long it would take for the ship to reach lightspeed and beyond.

I will also use this as a chance to explore negative gravity as that is part of the theoretical ship. Nothing about it has been physically proven, but it is mathematically possible. I find that area really cool as well and think that this could be a great experience in finding out more things about the field I want to go into.

Rationale:

My rationale for doing this is because I find it extremely interesting, it's a cool little thing that has a lot to do with the field I want to study in college which is Astro-Physics. I also am already interested in this particular subject of FTL travel because I have heard about it before and have studied it a bit on some occasions. I also want to know what kind of speeds would be possible and how long it would take for a machine like this to get to some neighboring stars. I think this is a very cool topic and think it could help me in the long run with my further education and career.

Some things to know:

m= Mass

E= Energy

G= Gravitational Constant ($6.674 \times 10^{-11} \text{ N} \cdot (\text{m}/\text{kg})^2$)

F= Force

r= Radius/ Distance between the two centers of masses of two objects effected by each other's gravity.

F_g= Force of gravity

m_s=Mass of Ship

m_{a1}= Mass of anomaly 1

m_{a2}= Mass of anomaly 2

Parameters:

Ok, so I'm going to assume the ship's mass as a constant beforehand so I don't have to mess around with it later, I will do the same for all the things I want to find, so I'm not looking for more things to do on the spot while writing later on.

The ship's mass can be twice the mass of a Saturn 5 rocket, because it would be bigger, and I think twice that mass is a good number to kind of estimate as it's impossible for us to know the true mass of a ship like this on account of it doesn't exist. A Saturn 5 is 2,970,000 kg, so twice that for our ships weight is 5,940,000 kg, so the mass of the ship, m_s= 5,940,000 kg

I will be testing first, where to place a_1 (anomaly 1) and a_2 (anomaly 2) around the front and back of ship for best pull/push through space. This should be fairly simple because I just make sure that it can't reach to the protected zone on the ship where space time is manipulated to be not wild and crazy so that there may be mortal passengers aboard and so the ship doesn't get karate chopped in two like a big Titanic being double teamed by 2 extremely powerful icebergs that are made out of manipulated space time. This safe zone is needed and is part of NASA's ship design because in a 2D rendering of Warped Spacetime, this could be like putting Mount. Everest and the Mariana Trench next to each other for a good old game of Tug o' War.

I will be testing so that a_1 has the mass of the earth, then sun, then just for funzies, the smallest black hole I can find, just to let thing really get wild. Then, all those values but negative for a_2 , which will be the force that pushes the ship instead of pulling it. I will see if a negative mass value works with Newton's equation, but I may have to tweak it a bit.

I will then be seeing how long it would take to:

- a) Reach Lightspeed with each of these values
- b) Reach the nearest star (Proxima Centauri) with these values
- c) See how long it would take travel across the Milky Way with these values

With my parameters set, I am ready to begin the tests and later come back if I need to add more variables or modify any of this a bit.

Where should I place the anomalies?

I want them at least like 100 or so meters out just for safety, but I still want them as close as possible for the strongest possible gravitational attraction. Because these are gravitational anomalies controlled by humans, its safe to say they wouldn't have to be a physical thing and instead would just be a warp in spacetime, so we don't need to worry

about collision with these ‘objects’ as if something fails, whatever is generating these anomalies shuts them down.

To set the positions, I’m going to get the highest F_g from the Newton gravity, and equation just for sake of simplicity, will use Earth’s mass with the ship’s so I can position it for a smaller mass to be a bit more realistic. Mass of Earth = $m_e = 5.9722 \times 10^{24}$ kg

Newton’s gravity equation will be sufficient for this paper, and I am not yet skilled in calculus so it would be very difficult to try to go into Einstein’s Field Equations, which I plan to learn about one day once I have furthered my understanding in calculus.

The gravity equation I’m using was formed by Newton and consists of a few parts. The final result in the equation is the Force of Gravity that the two objects have on each other, G is the gravitational constant, m1 and 2 are the 2 masses of the objects and r is the distance between the two centers of masses of the objects one would be studying. The distance between two objects was given the symbol r (for radius) because it made sense when calculating orbits and the gravitational pull on astronomical bodies but does sound a bit weirder for stuff, I’m doing like this.

$$F_g = G \frac{m_{a1}m_s}{r^2}, \quad G = 6.674 \times 10^{-11}$$

$$F_g = 6.674 \times 10^{-11} \frac{(5.9722 * 10^{24})(5,940,000)}{r^2}$$

$$r_1 = 100m \quad F_{g_{r_1}} = 6.674 \times 10^{-11} \frac{(5.9722 * 10^{24})(5,940,000)}{100^2}$$

$$F_{g_{r_1}} = 2.36759 * 10^{17}$$

$$r_2 = 10m \quad F_{g_{r_2}} = 6.674 \times 10^{-11} \frac{(5.9722 * 10^{24})(5,940,000)}{10^2}$$

$$F_{g_{r_2}} = 2.36759 * 10^{19}$$

$$r_3 = 50m \quad F_{g_{r_3}} = 6.674 \times 10^{-11} \frac{(5.9722 * 10^{24})(5,940,000)}{50^2}$$

$$F_{g_{r_3}} = 9.47037 * 10^{17}$$

Out of all these, I chose to go with r_1 as a nice distance to keep all of the tested anomalies at keeping in mind that the acceleration will happen infinitely, and I thought it would be the best for the imaginary safety of any imaginary humans riding in this imaginary space ship.

So, r is now a set constant for the rest of the paper at 100m. $r = 100m$

With a new constant for our mathematical tests decided, I am ready to begin the tests themselves. Test 1 will be gravitational anomalies with the mass of Earth and negative Earth, finding how quickly they reach lightspeed and the amount of time it would take for them to reach Proxima Centauri, then to travel across the Milky Way Galaxy.

Deriving an equation, I will use.

The equation,

$$v_x^2 = v_o^2 + 2a_s d$$

Is derived from a derivative:

$$\frac{dv}{ds} = ?$$

This is really weird, but it does equal itself, and itself times 1. You can write 1 is a fancy way to make something out of nothing basically, with a weird thing called “algebra with infinitesimals”. With those weird things, we can do this:

$$\frac{dv}{ds} = \frac{dv}{ds} 1$$

$$\frac{dv}{ds} = \frac{dv}{dt} \frac{dt}{ds}$$

$$\frac{dv}{ds} = \frac{dv}{dt} \frac{dt}{ds}$$

$$\frac{dv}{ds} = a \frac{1}{v}$$

Next, the variables need to be separated for integrals to come in and do their thing.

$$v \, dv = a \, ds$$

$$\int_{v_0}^v v \, dv = \int_{s_0}^s a \, ds$$

$$\frac{1}{2}(v^2 - v_0^2) = a(s - s_0)$$

$$v_x^2 = v_0^2 + 2a(s - s_0)$$

Or,

$$v_x^2 = v_0^2 + 2ad$$

For our purposes.

Test 1

Here are the variables that will be consistent with this test:

$$r_1 = 100\text{m}$$

$$r_2 = -100\text{m}$$

$$m_{a1} = 5.9722 \times 10^{24} \text{ kg}$$

$$m_{a2} = -5.9722 \times 10^{24} \text{ kg}$$

$$m_s = 5,940,000 \text{ kg}$$

$$G = 6.674 \times 10^{-11}$$

First we will use the gravity equation previously used to get the gravitational force between the two objects then we will add the forces from m_{a1} and m_{a2} to get a net force on the ship.

$$F_{g1} = G \frac{m_{a1}m_s}{r^2}$$

$$F_{g1} = 6.674 \times 10^{-11} \frac{(5.9722 \times 10^{24})(5,940,000)}{100^2}$$

$$F_{g1} = 2.36759 \times 10^{17}$$

$$F_{g2} = G \frac{m_{a2}m_s}{r^2}$$

$$F_{g2} = 6.674 \times 10^{-11} \frac{(-5.9722 \times 10^{24})(5,940,000)}{-100^2}$$

$$F_{g2} = 2.36759 \times 10^{17}$$

Now, because the force of m_{a2} is negative and is behind the ship, the r value we gave to it was also negative what the r value for the one in front of the ship was. This is because the ship is at 0m away from the ship. This will make it, so we are able to simply add the two F_g values we get and that is the net force pulling on the ship in the forwards direction.

$$F_N = F_{g1} + F_{g2}$$

$$F_N = 2.36759 \times 10^{17} + 2.36759 \times 10^{17}$$

F_{g1} = Gravitational Force
Between m_{a1} and m_s
 F_{g2} = Gravitational Force
Between m_{a2} and m_s
 F_N = Net Force of Gravity on
the Ship.

$$F_N = 4.73519 \times 10^{17}$$

From here, we may use the simple equation of $F=ma$ to determine the acceleration(a). For m , we can use m_s , because the m_1 isn't a real thing that needs to be moved, it's a (for us) magical future technology that manipulated space time itself, so it's not a physical object that needs to be moved, it's a concept that's position is always the same relative to the ship itself, so we don't have to worry about that, it doesn't really exist for this next part.

$$F_N = m_s a_s$$

F_N = Net Force of Gravity on the Ship
 m_s = Mass of the Ship
 a_s = Acceleration of the Ship

We want to extract the acceleration of this ship from this, and in order to do so, we need to rearrange the equation with simple algebra. This equation means that the net force in a system is equal to the mass times the acceleration, therefore an object at rest, or in equilibrium, will have no net force as it has no acceleration. The Ship will be infinitely accelerating, so it will have a constant force, that force for this test is the F_N we have already found, so now we can find the acceleration.

$$\frac{F_N}{m_s} = a_s$$

$$\frac{4.73519 \times 10^{17}}{5,940,000} = a_s$$

$$a_s = 7.97169 \times 10^{10} \text{ meters per second per second}$$

That's quite a lot of acceleration, at first I was expecting near to 9.8m/s/s because that's roughly the gravity on earth, so I expected it to be the same here, but then I remembered that for us on earth, that mass is distributed around and the distance from us to the earth's center of gravity is extremely large and far greater than 100m, so the point at which the ship is attracted to is much closer, so the total force is much greater because in the equation for gravity, the r value is squared, and its on the bottom, so the further

away the object/it's center of mass gets, the more you have to divide by so the total F_g gets weaker and weaker.

Now I will figure out how much time it will take for our ship to reach light speed with this constant acceleration. For this I need to use a new equation with a few more terms. It is a equation of motion, and it needs to be manipulated from its normal state to produce out time value.

$$v_x = v_o + a_s t$$

v_x = Final Velocity (speed of light) (c)
 v_o = Initial Velocity (0)
 a_s = Acceleration of ship
 t = Time spent accelerating

$$299,792,458 = 0 + 7.97169 \times 10^{10} t$$

$$\frac{299,792,458 \text{ m/s}}{7.97169 \times 10^{10} \text{ m/s}^2} = t$$

$$t = 0.003761 \text{ seconds}$$

So, we discover that we would reach lightspeed extremely quickly, in less than a second. This is far faster than I would have expected, but I guess it makes since, because the attractive thing is only 100m away, it would be like a black hole with earth's mass 100m away from you; as a result, you would accelerate towards it at an extremely rapid pace.

So, now I can find how long it would take for this machine to go various distances using another physics equation. Or, really two. We first need to find out what the final velocity will be at each of these distances, then we can find the time it would take with each of those. I will combine it all into one equation using substitution for ease later on.

First, I will calculate how long it would take to reach Proxima Centauri, so d_{pc} , will be the distance to Proxima Centauri, which is 4.014×10^{16} m. So first we start with the equation:

$$v_x^2 = v_o^2 + 2a_s d_{pc}$$

v_x = Final Velocity
 v_o = Initial Velocity
 a_s = Acceleration of the Ship
 d_{pc} = Distance to Proxima Centauri

$$v_x^2 = 0^2 + 2(7.97169 \times 10^{10})(4.014 \times 10^{16})$$

$$v_x^2 = 6.39967 \times 10^{27}$$

$$v_x = 7.9998 \times 10^{13}$$

Now, once again we can use the equation used to calculate how fast it would take to reach lightspeed with this new ending velocity to quickly get how long it would take to reach Proxima Centauri.

$$v_x = v_o + a_s t$$

$$7.9998 \times 10^{13} = 0 + 7.97169 \times 10^{10} t$$

$$\frac{7.9998 \times 10^{13} \text{ m/s}}{7.97169 \times 10^{10} \text{ m/s}^2} = t$$

$$t = 1003.53 \text{ seconds}$$

Or 16.7255 minutes.

v_x = Final Velocity to arrive at Proxima Centauri.
 v_o = Initial Velocity (0)
 a_s = Acceleration of ship
 t = Time to arrive

With that now answered, we can repeat all of that for the time it would take to go all the way across the Milky Way, but first, I'm going to make an equation that will make it easier to do it all in one calculation.

I can turn:

$$v_x^2 = v_o^2 + 2a_s d$$

Into:

$$v_x = \sqrt{v_o^2 + 2a_s d}$$

I can also turn:

$$v_x = v_o + a_s t$$

Into:

$$\frac{v_x - v_o}{a_s} = t$$

Then, using substitution, can put the first equation into the second so it looks like this:

$$\frac{\sqrt{v_o^2 + 2a_s d} - v_o}{a_s} = t$$

v_x = Final Velocity
 v_o = Initial Velocity (0)
 a_s = Acceleration of ship
 t = Time to arrive
 d = Distance

Now I can simply plug in the values and the new distance value for the Milky Way width which can be called d_{mw} , the value for d_{mw} turns out to be 1×10^{21} meters across, or at least that's our best estimate so far. So, here is how long it would take for this ship to go all the way across the Milky Way with two dimensional anomalies with the mass power of one earth and one negative earth.

$$\frac{\sqrt{v_o^2 + 2a_s d} - v_o}{a_s} = t$$

$$\frac{\sqrt{0 + 2(7.97169 \times 10^{10})(1 \times 10^{21})} - 0}{(7.97169 \times 10^{10})} = t$$

$$t = 158394 \text{ seconds}$$

$$\text{Or, } 43.9983 \text{ hours}$$

For **Test 1**, I tested how long it would take for a ship like this to reach light speed, travel to Proxima Centauri, and to travel all the way across the entire Milky Way galaxy all with the gravitational force of two anomalies on either side of the ship with a simulated mass of earth, and that of negative earth's mass. Also, with each of the anomalies located 100m away from the ship on both ends of it. It was shocking to me how quickly it would be able to make the journey. When I first saw how fast it would be

able to attain light speed I kind of laughed a bit because I thought it would be longer than a second.

Test 2

For this test, I will find how long it will take a ship with two space-time anomalies 100m in front of and behind it with the gravitational disturbance of 1 of the sun, and negative one of the sun, respectively. This should amount to a net force pulling the front of the ship by twice the force one sun would pull. The mass of the sun will be constant throughout this test and will be shown as m_{sun} in equations, this will be equal to 1.989×10^{30} kg.

Here are the things that will be constant throughout this test:

$$\begin{aligned} r_1 &= 100\text{m} \\ r_2 &= -100\text{m} \\ m_{a1} &= 1.989 \times 10^{30} \text{ kg} \\ m_{a2} &= -1.989 \times 10^{30} \text{ kg} \\ m_s &= 5,940,000 \text{ kg} \\ G &= 6.674 \times 10^{-11} \end{aligned}$$

First we must get the net force on the ship to determine the correct acceleration, we will do this the exact same way we did in Test 1.

$$F_{g1} = G \frac{m_{a1} m_s}{r^2}$$

$$F_{g1} = 6.674 \times 10^{-11} \frac{(1.989 \times 10^{30})(5,940,000)}{100^2}$$

$$F_{g1} = 7.8851 \times 10^{22}$$

$$F_{g2} = G \frac{m_{a2} m_s}{r^2}$$

$$F_{g2} = 6.674 \times 10^{-11} \frac{(-1.989 \times 10^{30})(5,940,000)}{-100^2}$$

F_{g1} = Gravitational Force
Between m_{a1} and m_s
 F_{g2} = Gravitational Force
Between m_{a2} and m_s
 F_N =Net Force of Gravity on
the Ship.

$$F_{g2} = 7.8851 \times 10^{22}$$

Now to get the net force on the ship we add these both together like before to see that the F_N on the ship is:

$$F_N = F_{g1} + F_{g2}$$

$$F_N = 7.8851 \times 10^{22} + 7.8851 \times 10^{22}$$

$$F_N = 1.57702 \times 10^{23}$$

To find the acceleration we do the same as before using $F=m_s a$, using the mass of the spaceship.

$$F_N = m_s a_s$$

$$\frac{F_N}{m_s} = a_s$$

F_N = Net Force of Gravity on the Ship
 m_s = Mass of the Ship
 a_s = Acceleration of the Ship

$$\frac{1.57702 \times 10^{23}}{5,940,000} = a_s = 2.65492 \times 10^{16}$$

With the acceleration found, I can find the time it will take to reach the speed of light using the first equation of motion I used in test 1:

$$v_x = v_o + a_s t$$

v_x = Final Velocity (speed of light) (c)
 v_o = Initial Velocity (0)
 a_s = Acceleration of ship
 t = Time spent accelerating

$$299,792,458 = 0 + 2.65492 \times 10^{16} t$$

$$\frac{299,792,458 \text{ m/s}}{2.65492 \times 10^{16} \text{ m/s}^2} = t$$

$$t = 1.1292 \times 10^{-8} \text{ seconds}$$

Or, .000000011292 seconds

This time around I was suspecting an insanely small number for this, and I got what I expected, this allows you to reach lightspeed so quickly its scary. It also makes me think, you probably don't want to turn the mass this far up near the solar system because it would definitely throw off the orbits of the other planets, and especially the sun. It would be advised to turn up the warped mass in front of you once you are in interstellar space far enough away from everything we care about so you don't accidentally destroy it all.

Now, we can continue by finding the amount of time it would take to reach the closest star system of Proxima Centauri. Using the equation we created at the end of test 1 to make things quicker and so that its easy to understand. The distance to Proxima Centauri again, is 4.014×10^{16} m

$$\frac{\sqrt{v_o^2 + 2a_s d_{pc}} - v_o}{a_s} = t$$

v_x = Final Velocity
 v_o = Initial Velocity (0)
 a_s = Acceleration of ship
 t = Time to arrive
 d_{pc} = Distance to Proxima Centauri

$$\frac{\sqrt{0 + 2(2.65492 \times 10^{16})(4.014 \times 10^{16})} - 0}{2.65492 \times 10^{16}} = t$$

$$t = 1.73891 \text{ seconds}$$

That's very fast, I'm kind of shocked by that one, but I guess I shouldn't have been, from what I know about the ratio of the earth's mass to the sun, this kind of fits in to what I should expect, its just for distances that people living now are unable to accomplish, its silly how fast we could one day be. Now for how long it would take to traverse across the whole galaxy, which again, is 1×10^{21} m.

$$\frac{\sqrt{v_o^2 + 2a_s d_{mw}} - v_o}{a_s} = t$$

v_x = Final Velocity
 v_o = Initial Velocity (0)
 a_s = Acceleration of ship
 t = Time to arrive
 d_{mw} = Diameter of the Milky Way

$$\frac{\sqrt{0 + 2(2.65492 \times 10^{16})(1 \times 10^{21})} - 0}{2.65492 \times 10^{16}} = t$$

$$t = 274.466 \text{ seconds}$$

Or 4.574433333 minutes

I'm also kind of shocked it would take that long, because that's a truly insane speed. Galactic colonization would be pretty easy if we actually were able to develop this technology and power it. It gives me a little bit of hope even though we will either die of a soon global nuclear war, or climate change. :3

Conclusion

In conclusion, the time it would take to conquer the stars and our own galaxy with this technology is truly astounding, The math I explored was physics based math, a lot of which, I used in AP Physics 1, IB Math SL 1 really helped me have a new look on all this stuff. In the final draft of this I will definitely clear up mistakes I'm sure happened, will make it more mathy if its not enough, might add in some more math from SL Math 1 if that needs to be done, and yea, I'm exited to work on the paper more making it the best it can be. Also, all of this is going to give me a big head start into understanding relativity, which is something I might add in the second draft to make it a higher level of math, but I just couldn't understand some of it this time as it requires an understanding of calculus I don't yet have.

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